

## Combining Astrometry and Spectroscopy

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**Abstract.** Orbital solutions for binary or multiple stellar systems that combine astrometry (e.g., position angles and angular separations) with spectroscopy (radial velocities) have important advantages over astrometric-only or spectroscopic-only solutions. In many cases they allow the determination of the absolute masses of the components, as well as the distance. Yet, these kinds of combined solutions that use different types of observations in a global least-squares fit are still not very common in the literature. An outline of the procedure is presented, along with examples to illustrate the sort of results that can be obtained. The same method can easily be extended to include other types of measurements (times of eclipse, Hipparcos observations, interferometric visibilities and closure phases, parallaxes, lunar occultations, etc.), which often complement each other and strengthen the solution.

### 1. Introduction

The main subject of this review is orbital solutions in binary and multiple stellar systems, with emphasis on bringing together observations from different techniques (mainly astrometry and spectroscopy) into a single fit that is usually better than separate solutions using either of those kinds of data. Orbit determination is a classical discipline in astronomy that dates back more than 170 years. The first astrometric orbit determination, based on measurements of the angular separation and position angle of the visual binary  $\xi$  UMa, is credited to Félix Savary (1827). The first determination of a spectroscopic orbit was made by Arthur A. Rambaut (1891), based on radial velocity measurements of  $\beta$  Aur by E. C. Pickering. Since then numerous algorithms have been developed for both kinds of solutions, which are well described in textbooks on the subject, and many thousands of orbits have been determined.

Because astrometric measurements describe motion on the plane of the sky, and spectroscopic measurements (radial velocities) describe motion along the line of sight, the two techniques are complementary and it is fairly obvious that there are advantages in combining them for the same system. This can easily be seen by looking at the classical orbital elements used to describe each type of orbit. For astrometric orbits of binaries the elements are  $P$ ,  $a''$ ,  $e$ ,  $i$ ,  $\omega$ ,  $\Omega$ , and  $T$ , which have their usual meaning, and where we indicate by  $a''$  the *angular* semimajor axis, to distinguish it from the *linear* semimajor axis that appears as a derived quantity in spectroscopy (see below). The conventional

spectroscopic elements for a double-lined spectroscopic binary are  $P$ ,  $\gamma$ ,  $K_1$ ,  $K_2$ ,  $e$ ,  $\omega$ , and  $T$ , with the usual caveat that  $\omega$  here is the longitude of periastron for the primary component, whereas the normal convention in visual orbits is to use the longitude of periastron for the secondary component. Trivially the two angles differ by  $180^\circ$ . Four orbital elements are in common between astrometric and spectroscopic solutions:  $P$ ,  $e$ ,  $\omega$ , and  $T$ , which means that both kinds of observations can constrain them.

## 2. Combined solutions: Advantages and disadvantages

Having astrometric and spectroscopic observations available for the same system allows a much more complete description of the path of the stars in space (sometimes referred to as a “three-dimensional orbit”). In addition, new information can be obtained by combining the two kinds of data, most notably the individual masses of the components (in double-lined astrometric-spectroscopic binaries), and the “orbital parallax”. The latter allows a direct determination of the luminosities of the stars.

The absolute masses in a binary follow from the expressions for the minimum mass derived from the spectroscopic solution,

$$\begin{aligned} M_1 \sin^3 i &= P(1 - e^2)^{3/2} (K_1 + K_2)^2 K_2 \\ M_2 \sin^3 i &= P(1 - e^2)^{3/2} (K_1 + K_2)^2 K_1, \end{aligned}$$

and the inclination angle  $i$  provided by astrometry.

The orbital parallax is a direct measure of the distance free from any assumptions beyond Newtonian physics, and can often be more precise than trigonometric parallaxes, particularly for more distant systems. It follows trivially from the ratio between the projected angular semimajor axis from astrometry ( $a'' \sin i$ ), and the projected linear semimajor axis from spectroscopy,

$$a \sin i = P(K_1 + K_2) \sqrt{1 - e^2},$$

as

$$\pi_{\text{orb}} = \frac{a'' \sin i}{P(K_1 + K_2) \sqrt{1 - e^2}}.$$

If we are interested in combining the two different types of data, the question then arises as to how to do this in the optimal way. One possibility, which is by far the most common approach seen in the literature, is simply to carry out completely separate astrometric and spectroscopic fits if possible, and proceed to use the information provided by these separate solutions to derive other properties of the system (such as  $M_1$ ,  $M_2$ , and  $\pi_{\text{orb}}$ ). In this approach the four elements in common between the two types of solutions ( $P$ ,  $e$ ,  $\omega$ , and  $T$ ) are usually averaged, possibly with some weighting, whereas the elements  $a''$ ,  $i$ , and  $\Omega$  are adopted from the visual solution, and  $\gamma$ ,  $K_1$  and  $K_2$  are taken directly from the spectroscopy. One disadvantage of this approach is that separate spectroscopic and astrometric solutions are not always possible for a given system. This can happen, for instance, when the phase coverage of one type of observation

is insufficient for an independent fit, even though the other type of observation may allow a good solution. In addition, by averaging the four elements in common, the uncertainties to be assigned to those elements are ill-defined and often not very realistic. Separate solutions ignore the redundancy provided by the measurements, and are wasteful of information that is sometimes critical to avoid systematic errors.

The second approach to a combined astrometric-spectroscopic solution is to merge all the data into a single least-squares fit, instead of having two separate fits. In this case one solves simultaneously for all of the orbital elements (10 in the case considered above, of a double-lined spectroscopic system that is also spatially resolved). An important advantage of this method is that combined (three-dimensional) solutions are often possible even if one type of observation (astrometry or spectroscopy) has insufficient coverage for an independent fit. In some cases solutions are possible when neither the astrometric or the spectroscopic observations are sufficient by themselves. In general these simultaneous fits strengthen the determination of the elements because they fully exploit the constraints available from both types of data (redundancy). In addition, this redundancy allows for useful checks of systematic errors. Furthermore, the errors in the elements are straightforward to derive because they come from a single least-squares fit, and do not suffer from the ambiguities mentioned above (particularly for  $P$ ,  $e$ ,  $\omega$ , and  $T$ ).

Perhaps the only drawback is that combined solutions are perceived by astronomers to be somewhat more complex mathematically, although this not necessarily true, as we describe below.

### 2.1. The mathematics of combined orbital solutions

The way in which spectroscopic or astrometric least-squares solutions are usually posed is well known and conceptually quite simple: the “best-fit” elements are those that minimize the sum of the normalized residuals squared, or  $\chi^2$ . The normalization is achieved by dividing the residuals by the uncertainty of each measurement, which provides the weighting. Thus, for an astrometric solution in which the measurements are the angular separations ( $\rho$ ) and position angles ( $\theta$ ), with uncertainties  $\sigma_\rho$  and  $\sigma_\theta$ , respectively, the expression to be minimized is

$$\chi_{astr}^2 = \sum \left( \frac{\rho - \rho^*}{\sigma_\rho} \right)^2 + \sum \left( \frac{\theta - \theta^*}{\sigma_\theta} \right)^2, \quad (1)$$

where  $\rho^*$  and  $\theta^*$  are the predicted values based on the orbital elements at each iteration, and the sums are carried out over all observations of each kind. If the measurements are reported in rectangular rather than polar coordinates, such as those sometimes made with CCDs or photographic plates, then  $x$  and  $y$  and their corresponding errors should be used instead of  $\rho$  and  $\theta$ .

An analogous expression holds for a spectroscopic solution in which the velocities  $RV_1$  and  $RV_2$  of both components are measured, which is

$$\chi_{spec}^2 = \sum \left( \frac{RV_1 - RV_1^*}{\sigma_{RV_1}} \right)^2 + \sum \left( \frac{RV_2 - RV_2^*}{\sigma_{RV_2}} \right)^2, \quad (2)$$

where  $RV_1^*$  and  $RV_2^*$  are again computed at each iteration from the orbital elements.

The numerical problem of solving for the orbital elements involves non-linear equations in both cases, but techniques for tackling such cases are readily available and are in common use, such as the Levenberg-Marquardt algorithm (see, e.g., Press et al. 1992), and others.

In a solution that combines astrometric and spectroscopic measurements, the figure of merit to be minimized is simply the sum of the separate  $\chi^2$  values, or

$$\chi_{comb}^2 = \chi_{astr}^2 + \chi_{spec}^2. \quad (3)$$

As trivial as the above equation may seem, the problem of combining observations of different kinds to derive a three-dimensional orbit was first formulated in this way less than 30 years ago by C. Morbey at the Dominion Astrophysical Observatory (Morbey 1975). The author applied it to the case of the highly eccentric visual binary system Burnham 1163 (ADS 1123, HD 8556), which had good astrometric coverage at relatively low precision and spectroscopic coverage only during periastron, but with much better precision.

Thus, the mathematics of a combined solution are no more complex than those of either of the separate solutions, and identical numerical techniques can be used.

### 3. Applications

One of the most obvious applications of combined astrometric-spectroscopic solutions is the determination of accurate stellar masses and orbital parallaxes. The parallaxes along with the apparent brightness of each component allow the luminosities to be derived. Masses and luminosities are fundamental data of great importance for testing models of stellar evolution, particularly in some regimes where the models are still poorly constrained, such as the lower main sequence (see, e.g., Henry & McCarthy 1993; Henry et al. 1999).

Systems with multiplicity higher than two (triples, quadruples) often benefit from the simultaneous availability of astrometric and radial-velocity measurements, and allow the masses of all of the stars in the system to be determined. These cases are also of interest for investigating the coplanarity of inner and outer orbits in hierarchical configurations.

Another interesting recent application in which spectroscopic and astrometric information are combined to great advantage is the determination of masses for substellar objects (brown dwarfs and extrasolar planets). Such basic information is almost non-existent for such objects, yet it is crucial to confront the theories that are being developed for their formation and evolution.

In the following we describe several specific examples where combined astrometric-spectroscopic solutions have been carried out, and we also extend the idea to other types of measurement.

#### 3.1. The mass-luminosity relation in the Hyades

Mass determinations in stellar clusters are particularly interesting for testing models of stellar evolution because of the additional constraints that are available: cluster members can all be assumed to have the same age and chemical composition, and in many cases these are well known from spectroscopic studies

and isochrone fits to the color-magnitude diagram. The Hyades is a well-known and particularly important example for this kind of study. Five binary systems in this cluster have absolute mass determinations, four of which come from combined astrometric-spectroscopic solutions (51 Tau, 70 Tau,  $\theta^1$  Tau, and  $\theta^2$  Tau; Torres, Stefanik, & Latham 1997a;b;c). The other system (V818 Tau) is an eclipsing binary. The observational data used for the former 4 systems include visual measurements (made with filar micrometers or eyepiece interferometers), measurements by speckle interferometry, lunar occultation, and long-baseline interferometry, as well as radial velocity measurements for one or both stars in the binary. The angular sizes of the orbits range from 19 milli-arcseconds ( $\theta^2$  Tau,  $P = 140.7$  days) to  $0''.13$  ( $\theta^1$  Tau,  $P = 16.3$  yr). In the case of  $\theta^1$  Tau, which is only a single-lined spectroscopic binary (the primary star is a giant and the secondary is a main-sequence star), the distance was used in order to derive the component masses in the combined solution. For  $\theta^2$  Tau the spectroscopic elements were combined directly with the elements derived from long-baseline interferometry, since the original astrometric observations were not available.

The absolute masses of the 9 main-sequence stars in these binary systems were used to construct the empirical mass-luminosity relation in the Hyades, and to compare it with model isochrones computed specifically for the age and composition of the cluster. Subtle differences were found, possibly indicating a helium abundance different from solar. In addition, the orbital parallaxes derived for 51 Tau, 70 Tau, and  $\theta^2$  Tau, slightly more precise than those from the Hipparcos mission, served as a stringent test for systematics in the satellite determinations and are mentioned on the Hipparcos web site as a valuable external check (see also de Bruijne, Hoogerwerf, & de Zeeuw 2001).

### 3.2. Systems of higher multiplicity

An interesting example of the power of combined astrometric-spectroscopic solutions in higher multiplicity configurations is given by the quadruple system  $\mu$  Ori, studied by Fekel et al. (2002). This is a visual binary with a highly eccentric orbit and a semimajor axis of  $0''.27$  ( $P = 18.6$  yr), in which one component is itself a double-lined spectroscopic binary with a period of  $P = 4.78$  days and the other component is also a spectroscopic binary (but only single-lined) with a similar period of  $P = 4.45$  days. Visual and speckle observations of the visual pair were combined with radial-velocity measurements of the 3 visible objects, and the authors were able to derive the absolute masses of the stars in the double-lined binary with relative errors of only 2%. An orbital parallax was also derived for the system, which is more than 4 times more precise than the Hipparcos parallax, but is in good agreement with it.

### 3.3. Masses for substellar objects

In recent years dozens of unseen companions to solar-type stars have been detected by means of highly-precise radial velocity measurements. From their small minimum masses these companions appear to be planetary in nature, and the subject has attracted great attention not only for studies of the origin and evolution of our solar system, but also because of its implications for the possibility of extraterrestrial life. But since Doppler spectroscopy only provides a lower limit to the mass of such objects, bringing in complementary astrometric

information is an important application of the techniques described above. The case of Gl 876 is a good illustration. It is an M4 dwarf star with two presumably substellar companions in orbit with periods of about 30 days and 60 days. The star was observed by Benedict et al. (2002) with the Fine Guidance Sensors aboard the Hubble Space Telescope, and the wobble of the star due to the outer companion was detected. The authors combined their astrometric observations with existing radial velocities and solved for the parallax, proper motion, and the orbital elements of the relative orbit, which has a semimajor axis of only  $0.25 \pm 0.06$  mas. Their determination of the inclination angle ( $84^\circ \pm 6^\circ$ ) allowed them to establish the absolute mass of the orbiting companion at  $1.89 \pm 0.34$  times the mass of Jupiter, thus showing conclusively that it is a planet. This is the first astrometrically determined mass of an extrasolar planet.

Among the more than 100 low-mass companions detected by the Doppler planet searches, a few of the more massive ones with minimum masses between about 10 and 65 Jupiter masses were investigated by Halbwachs et al. (2000) to attempt to determine their true masses by establishing the inclination angles astrometrically. If the masses were to come out lower than 80 Jupiter masses (the substellar limit), they would fall in the “brown dwarf desert”, a term used to refer to the apparent lack of brown dwarf companions with relatively short periods in Doppler planet surveys of solar type stars. Hipparcos intermediate data (abscissa residuals) were combined with the spectroscopic orbital elements for 11 candidate brown dwarfs, but most of them were shown to have true masses above the substellar limit, or only slightly below but with relatively low confidence. Similar solutions were attempted later by other investigators to determine the inclination angles (and therefore the masses) of even lower-mass (planetary) companions, but those results were shown by Pourbaix (2001), Zucker & Mazeh (2001), and Pourbaix & Arenou (2001) to be spurious due to numerical reasons (see also the article by D. Pourbaix in this volume).

#### 4. Combined solutions of other kinds

In the applications described above spectroscopic information was combined with astrometric measurements, although the latter are one-dimensional abscissa residuals provided by the Hipparcos mission (in milli-arcseconds, along a reference great circle), rather than traditional two-dimensional measurements such as  $\{\rho, \theta\}$  or  $\{x, y\}$ . Operationally, however, the procedure is similar to the astrometric-spectroscopic solutions described earlier. In fact, the idea of combining measurements of different types is not limited to astrometry+spectroscopy. As an example, spectroscopic observations can be combined with eclipse timings measured for eclipsing binaries, based on photometry. The next section illustrates this with some examples.

##### 4.1. Solutions using times of eclipse

The orbital periods of eclipsing binaries are often determined from the accurate measurement of times of minimum light (for the primary and/or secondary eclipse). The period and epoch are usually then fixed for the spectroscopic solution. There are situations, however, where a combined fit that incorporates the eclipse timings together with the radial velocities has significant advantages,

such as when the times of minimum and the velocities are separated by a significant interval of time. In that case combining them gives a much longer time baseline. Mathematically the eclipse timings are incorporated into the solution by simply adding the  $\chi^2$  term

$$\chi_{time}^2 = \sum \left( \frac{T_I - T_I^*}{\sigma_{T_I}} \right)^2 + \sum \left( \frac{T_{II} - T_{II}^*}{\sigma_{T_{II}}} \right)^2, \quad (4)$$

to the spectroscopic term in eq.(2), so that the total  $\chi^2$  is  $\chi_{comb}^2 = \chi_{spec}^2 + \chi_{time}^2$ . The symbols  $T_I^*$  and  $T_{II}^*$  above are the predicted times of eclipse for the primary and secondary, based on the orbital elements at each iteration. Examples of binaries where this has been applied are FS Mon (Lacy et al. 2000) and EI Cep (Torres et al. 2000a).

A number of eclipsing binaries present apsidal motion (a change in  $\omega$ ), which over time alters the shape of the radial velocity curves. Such cases provide another application where the combination of eclipse timings and velocities into a single least-squares solution can significantly strengthen the determination of the apsidal motion constant ( $d\omega/dt$ ). This quantity contains information on the internal structure of stars (specifically, their degree of mass concentration), and as such it can provide important constraints to the theory of stellar interiors. V364 Lac (Torres et al. 1999) and GG Ori (Torres et al. 2000b) are two cases where this approach has been followed.

The types of combined solutions that have been mentioned so far merge together astrometry and radial velocity information, or astrometry and eclipse timings. Although one might not think such a case would ever arise in practice, the combination of astrometry and eclipse timings is also possible in the system R CMa, studied by Ribas, Arenou, & Guinan (2002). This is an Algol-type eclipsing binary ( $P = 1.14$  days) where the residuals from the recorded times of eclipse that span more than a century show a significant modulation due to the light-travel time effect. Additionally, Hipparcos measurements reveal small acceleration terms (non-linear proper motions). Both of these effects are nicely explained by the presence of a third star in a distant orbit with a period of about 93 yr. By incorporating also ground-based astrometric measurements to constrain the proper motion, the parameters of the outer orbit can be derived through a global solution, and its orientation turns out to be consistent with being coplanar with the inner orbit.

#### 4.2. Alternative astrometric-spectroscopic solutions

Spectroscopic observations can be incorporated into combined solutions in ways other than by using the radial velocities. Forveille et al. (1999) applied this technique to the astrometric-spectroscopic binary Gl 570BC, a pair of M dwarfs resolved by adaptive optics measurements as well as by 1-D and 2-D speckle interferometry. Instead of using velocities, they incorporated the cross-correlation profiles directly into the solution, adding to the usual orbital elements 4 new parameters that describe the Gaussian correlation peaks for the two components. Significant numerical advantages and improved errors are claimed with this technique, as well as less sensitivity to systematic errors in the velocity amplitudes. The main disadvantage is that there is no radial-velocity curve to admire, because radial velocities are bypassed altogether. Additional examples

of this approach include Gl 234, Gl 747, Gl 831, and Gl 866 (Ségransan et al. 2000).

Finally, astrometric information can also be incorporated into combined solutions without directly involving positional measurements on the sky such as  $\{\rho, \theta\}$ . This is the case, for example, in long-baseline interferometry where the measurement is the “fringe visibility”, which represents the contrast of the interference fringes. Visibilities (often used squared, as  $V^2$ ) implicitly contain information on the relative position of the components (see, e.g., Boden et al. 2000), and can be treated as any other observable by writing the corresponding  $\chi^2$  term

$$\chi_{astr}^2 = \sum \left( \frac{V^2 - (V^2)^*}{\sigma_{V^2}} \right)^2 \quad (5)$$

and combining it with the spectroscopic term in eq.(2). This has a number of important advantages over the alternative approach, which would be to derive  $\{\rho, \theta\}$  based on the fringe measurements on each night. The latter is not always possible if there are only a few visibilities on a given night, whereas the individual visibilities can be incorporated directly as indicated in eq.(5) even if there is only one measurement on that night. In addition, using the visibilities directly allows one to account for motion even within a night for short-period systems. This technique has been applied to Capella (Hummel et al. 1994), 12 Boo (Boden et al. 2000), HD 195987 (Torres et al. 2002), and many other binary systems.

## 5. Conclusions

Simultaneous astrometric-spectroscopic solutions usually have important advantages compared to separate fits to the astrometry or the radial velocities. These types of combined fits are still not seen very often in the literature, but with advances in observational techniques and increased access to data, they should become more common. The same approach can be extended to many other types of observations aside from  $\{\rho, \theta\}$  and velocities, such as Hipparcos intermediate data (abscissa residuals or transit data), lunar occultations, interferometric visibilities and closure phases, parallaxes, positions and proper motions, cross-correlation profiles, times of eclipse, and even magnitudes.

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